POLYGONAL REMAINDER PATTERNS

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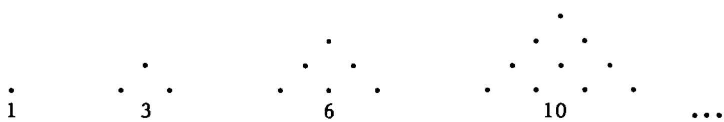
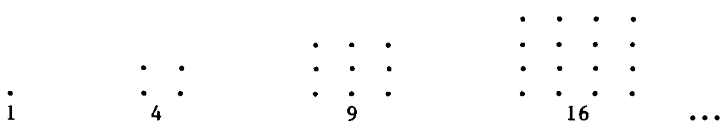
Teachers and their students frequently wish to use activities which provide for drill and practice and also encourage the review or discovery of number patterns. A rich source for such activities is the set of polygonal numbers.

The two most familiar sets of polygonal numbers are the triangular and square numbers. The triangular numbers (1, 3, 6, 10, ...) are so named because, after the first number, they represent numbers of points that can be arranged in triangular shapes as shown below:



Square numbers similarly represent points that can be arranged in the shapes of squares:



This concept of polygonal numbers can be extended to polygons of greater numbers of slides; some of these polygonal numbers are displayed below:

Polygonal numbers

Naturals	1	2	3	4	5	6	7	8	9	10	...
Triangular	1	3	6	10	15	21	28	36	45	55	...
Squares	1	4	9	16	25	36	49	64	81	100	...
Pentagonal	1	5	12	22	35	51	70	92	117	145	...
Hexagonal	1	6	15	28	45	66	91	120	153	190	...
Heptagonal	1	7	18	34	55	81	112	148	189	235	...
Octagonal	1	8	21	40	65	96	133	176	225	280	...
Nonagonal	1	9	24	46	75	111	154	204	261	325	...
Decagonal	1	10	27	52	85	126	175	232	197	370	...

We now examine a division pattern involving these polygonal numbers.

Activity 1:

Divide each triangular number by three and retain the remainder:

$1 \div 3$ leaves a remainder of 1
$3 \div 3$ leaves a remainder of 0
$6 \div 3$ leaves a remainder of 0
$10 \div 3$ leaves a remainder of 1
$15 \div 3$ leaves a remainder of 0
$21 \div 3$ leaves a remainder of 0

Verify that this 1, 0, 0, 1, 0, 0, ... pattern continues.

Activity 2:

Divide each square number by four and retain the remainder:

$1 \div 4$ leaves a remainder of 1
$4 \div 4$ leaves a remainder of 0
$9 \div 4$ leaves a remainder of 1
$16 \div 4$ leaves a remainder of 0
$25 \div 4$ leaves a remainder of 1
$36 \div 4$ leaves a remainder of 0

$49 \div 4$ leaves a remainder of 1

$64 \div 4$ leaves a remainder of 0

Verify that this 1, 0, 1, 0, 1, 0, 1, 0, ... pattern continues.

Activity 3:

Divide each pentagonal number by five and retain the remainder:

$1 \div 5$ leaves a remainder of 1

$5 \div 5$ leaves a remainder of 0

$12 \div 5$ leaves a remainder of 2

$22 \div 5$ leaves a remainder of 2

$35 \div 5$ leaves a remainder of 0

$51 \div 5$ leaves a remainder of 1

$70 \div 5$ leaves a remainder of 0

$92 \div 5$ leaves a remainder of 2

$117 \div 5$ leaves a remainder of 2

$145 \div 5$ leaves a remainder of 0

Verify that this 1, 0, 2, 2, 0, 1, 0, 2, 2, 0, ... pattern continues.

Activity 4:

Observe that for n = 3, 4, 5 the division of n-gonal numbers by n led to a sequence of remainders which could be grouped into sets of n-numbers which repeated indefinitely. Does this pattern continue for n = 6, 7, 8, ...? If so, what patterns result? Try it and see!